MODEL PREDICTIVE CONTROL OF SURFACE PERMANENT MAGNET SYNCHRONOUS MOTORS

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Abstract: Synchronous machines became more popular in many industrial applications due to the progress of magnetic materials permanent magnet. Classical vector control structure is used to control synchronous drives. This paper presents alternative structure to control synchronous drives. Model Predictive Control - MPC is based on calculation of optimal trajectory according to the cost function. Application of explicit and on-line MPC is described.

Keywords: SPMSM drive, model predictive control, cost function, linearization

1 INTRODUCTION

Permanent magnet synchronous motor are nowadays used more often in industrial applications than before. In most applications standard control algorithms are used. However, the biggest problems of the standard structure are wind-ups and inefficient work with voltage, current and power constrains. Complicated field weakening implementation is their big disadvantage.

Therefor, new algorithms are applied to control the synchronous drives. One of the discussed algorithms is Model Predictive Control(MPC). The advantage of MPC is very efficient work with constrain. MPC can also handle field weakening tasks. However, the computation time is usually very long due to the complicated optimization problem solution.

2 MPC CONTROL

Design of MPC controller can be summarized in three steps. In the first step model of the controlled system is defined. In the second step, the cost function and the length of prediction horizon are specified and in the last step the optimal trajectory is calculated.

Model in MPC should predict the response of the system to the applied signal on the input of the system. In most cases model is linear because many optimizations methods for linear system are known. Nonlinear optimization is a very complicated task therefor nonlinear systems are usually tried to be linearized for the MPC task.

SPMSM is a nonlinear system [1]. State space discrete model can be defined as:

$$
\begin{aligned}
i_{\mathrm{d}}(k+1) &= i_{\mathrm{d}}(k) - \frac{\mathrm{T_s R}}{\mathrm{L}} i_{\mathrm{d}}(k) + \mathrm{T_s}\omega(k) i_{\mathrm{q}}(k) + \frac{\mathrm{T_s}}{\mathrm{L}} u_{\mathrm{d}}(k) \\
i_{\mathrm{q}}(k+1) &= i_{\mathrm{q}}(k) - \frac{\mathrm{T_s R}}{\mathrm{L}} i_{\mathrm{q}}(k) - \mathrm{T_s}\omega(k) i_{\mathrm{d}}(k) - \frac{\mathrm{T_s K_E}}{\mathrm{L}}\omega(k) + \frac{\mathrm{T_s}}{\mathrm{L}} u_{\mathrm{q}}(k) \\
\omega(k+1) &= \omega(k) + \mathrm{T_s}\frac{3}{2\mathrm{J}}\mathrm{p_p}^2 \mathrm{K_E} i_{\mathrm{q}}(k)
\end{aligned}
\tag{1}
$$

Model of the system allows to predict system's behavior. Prediction horizon is chosen. Prediction horizon describes how far the regulator calculates the trajectory. Large prediction horizon makes the regulator very precise. However, complexity of the problem grows exponentially with the enlargement of the prediction horizon.

The cost function sets up the control objectives [2]. The cost function is usually in the form of quadratic weighted criteria:

$$\mathrm{J}\left(\mathbf{u}|\mathbf{x}\left(t_0\right),t_0\right)=\sum_{i=0}^{N}\mathbf{x}^T\left(t_0+t_i|t_0\right)\mathbf{Q}\,\mathbf{x}\left(t_0+t_i|t_0\right)+\sum_{i=0}^{N_u-1}\mathbf{u}^T\left(t_0+\tau_i|t_0\right)\mathbf{R}\,\mathbf{u}\left(t_0+\tau_i|t_0\right) \tag{2}$$

Minimum of the criterion (2) is searched according to initial state x_0. Optimal control sequence u is found but only he first calculated control is used and the rest is discarded. In the next step the optimization is calculated again. This is called the receding horizon idea.

Quadratic programming is usually used to calculate the optimization task. There are two basic methods to calculate the optimization task - active set approach and interior point approach. When quadratic programming is used the state, input and output constrains can be defined. However, the constrains are to be defined in linear form. If quadratic constrain are used quadratic programming cannot be used unless appropriate linearization of the problem are found.

There are two basic points of view to the problem solution. On-line MPC and explicit MPC. The idea of on-line MPC is just to define problem and solve the optimization problem in every step. The idea of explicit MPC is that the control law is calculated in advance. The state space is divided in similar cones and the optimal control is calculated. The optimization problem is not solved every step, the precalculated input is used according to the rightful state.

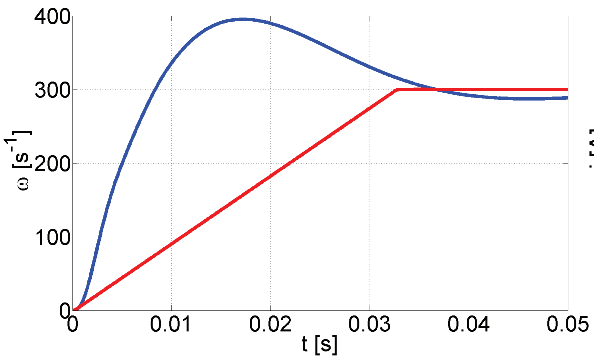
3 SIMULATIONS

SPMSM is nonlinear therefore the model needs to be linearized. There are only two nonlinearities in the motor(1) - ωi_d and ωi_q. The model can be linearized by definition two new extra state variables. It will not cause problems to MPC, because the real state is measured before the optimal trajectory is calculated. The only think to be defined is the way how those nonlinearities change in the prediction horizon.

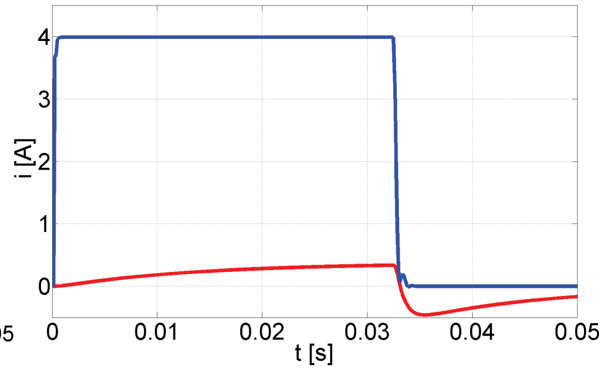
Usual constrains for the motors are in quadratic form - $u_d^2+u_q^2<\mathrm{U_{max}}^2$. Standard linear solvers cannot work with this description of constrains. All the constrains must be in linear form. If new state variable is defined to describe the quadratic constrain $u_{quad}=u_d^2+u_q^2$ then new constrain can be written as $u_{quad}<\mathrm{U_{max}}^2$. The larges disadvantage of this solution is enlargement of the state space. From former three state variables the state space model is enlarged to seven state variables.

$$\begin{aligned}
i_\mathrm{d}(k+1) &= i_\mathrm{d}(k)-\frac{\mathrm{T_sR}}{\mathrm{L}}i_\mathrm{d}(k)+\mathrm{T_s}\omega i_\mathrm{q}(k)+\frac{\mathrm{T_s}}{\mathrm{L}}u_\mathrm{d}(k)\\
i_\mathrm{q}(k+1) &= i_\mathrm{q}(k)-\frac{\mathrm{T_sR}}{\mathrm{L}}i_\mathrm{q}(k)-\mathrm{T_s}\omega i_\mathrm{d}(k)-\frac{\mathrm{T_sK_E}}{\mathrm{L}}\omega(k)+\frac{\mathrm{T_s}}{\mathrm{L}}u_\mathrm{q}(k)\\
\omega(k+1) &= \omega(k)+\mathrm{T_s}\frac{3}{2\mathrm{J}}\mathrm{p_p}^2\mathrm{K_E}i_\mathrm{q}(k)\\
\omega i_\mathrm{d}(k+1) &= \omega i_\mathrm{d}(k)+\omega_{nom}\frac{\mathrm{T_s}}{\mathrm{R}}u_d(k)\\
\omega i_\mathrm{q}(k+1) &= \omega i_\mathrm{q}(k)+\omega_{nom}\frac{\mathrm{T_s}}{\mathrm{R}}u_q(k)\\
u_{quad}(k+1) &= u_{quad}(k)+2u_\mathrm{d}(k)+2u_\mathrm{q}(k)\\
i_{quad}(k+1) &= i_{quad}(k)+2\frac{1}{\mathrm{R}}u_\mathrm{d}(k)+2\frac{1}{\mathrm{R}}u_\mathrm{q}(k)
\end{aligned} \tag{3}$$

Application of the on-line MPC is easier. Jacobi matrix is used to describe of the system and is updated in every step. In comparison with explicit MPC, on-line MPC is more efficient. The calculation takes longer.



(a) Comparison of classical vector control structure (blue) with MPC (red)



(b) Response of rotor currents i_d(red) and i_q(blue), MPC control

Figure 1: Simulations

Comparison of classical vector control structure with MPC can be seen in the figure 1a. Maximum momentum is used to reach desired speed and no overshoot appears in the MPC step response. Systems constrains are strictly kept (figure 1b).

4 CONCLUSION

MPC is generally very efficient tool for regulation. Its treatment of constrains is of very high quality. The main problem of MPC are extremely high computational demands that the optimization problem is solved on time.

When the model of SPMSM is linearized we can apply MPC on it. Both explicit and on-line MPC can be used. The explicit MPC cannot be very well linearized for the whole state space. However its advantage is faster computation. On-line MPC, on the other hand, takes very long to calculate but it can much better work with the constrain and is more precise.

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REFERENCES

[1] Caha, Z., Černý, M.: Elektrické pohony, Praha, SNTL, 1990, ISBN 80-03-00418-7

[2] Rawlings, J., Mayne, D.: Model Predictive Control: Theory and Design, Nob Hill Publishing, LLC, 2009, ISBN 978-0-97593377-0-9

[3] Štecha, J., Pekař, J.: Short Course on Model Predictive Control, Brno, Centrum pro rozvoj výzkumu pokročilých řídicích a senzorických technologií, 2009, Available from WWW: <http://crr.vutbr.cz/system/files/brozura_07_1005.pdf>